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RESEARCH MEMORANDUM

THE EFFECT OF VARIOUS MISSILE CHARACTERISTICS ON AIRFRAME FREQUENCY RESPONSE

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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THE EFFECT OF VARIOUS MISSILE CHARACTERISTICS ON AIRFRAME FREQUENCY RESPONSE

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SUMMARY

The frequency response of two important airframe characteristics were calculated for a cruciform, variable-incidence, air-to-air missile. Plots of amplitude ratio and phase angle as a function of angular frequency are presented to show the effects of the importance, accuracy, and the nonlinearity of certain aerodynamic derivatives, altitude and Mach number variation, and aeroelasticity. Altitude changes were found to have predominantly the greatest effect on the frequency response, while possible errors in the evaluation of the significant aerodynamic derivatives resulted in the least change.

INTRODUCTION

As is well known, the response of a missile to control motions depends on the mass characteristics, the flight altitude and speed, and the magnitude of the aerodynamic derivatives. As is well known, a number of these derivatives cannot be estimated with the desired accuracy. In addition, certain aerodynamic characteristics may not be linear, the missile may be subject to aeroelastic effects, and the missile may be required to operate over wide ranges of altitude and Mach number. It is the purpose of this paper to present and compare the effects of a number of these factors on several of the more important airframe-response characteristics. It was believed that such an investigation would give a better understanding of the problems of designers who must try to cope with these changes in one manner or another to obtain a satisfactory system response over the missile operating range. Since these changes appear to be particularly severe for a maneuvering type of missile, the typical short-range, air-to-air, boost-glide missile shown in figure 1 has been selected as an example, and the results which are presented are based on the mass and moment of inertia being assumed constant.

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THE FREQUENCY RESPONSE

The study is confined to longitudinal motion, and the airframe characteristics selected for scrutiny are shown in figure 2 and are the rate of change of the pitch angle θ and the rate of change of the flight-path angle γ in response to a control deflection δ. If a sine wave motion of the control is applied at a number of discrete frequencies, the relationship between the steady sinusoidal motion of the output such as the rates of change of θ and γ to the input is termed the frequency response. The frequency response is particularly useful in the design of the guidance and stabilization system, and therefore this form will be used in presenting the results. The location of the quantities $\dot{\theta}/\delta$ and $\dot{\gamma}/\delta$ in a portion of a typical guidance and stabilization system is shown in block-diagram form in figure 3. The quantity $\dot{\theta}/\delta$ was chosen because of its importance in the stabilization of the missile, while $\dot{\gamma}/\delta$ is a particularly significant relationship in the guidance loop of a proportional-navigation guidance system.

FREQUENCY-RESPONSE PLOTS

Relative Importance of the Aerodynamic Derivatives

First the relative importance of the aerodynamic derivatives on the frequency response is considered. Figure 4 shows the system of axes, the three equations of motion for constant thrust, and the derivatives used in these equations. The large number of the derivatives and the relative accuracy with which some of them can be evaluated leads to two questions; first, which derivatives have a significant effect on the results and, second, to what accuracy must the important derivatives be estimated to insure an adequate approximation of the frequency response. To study this, the general expressions relating the quantities of interest were derived from the equations of motion in the usual manner. The frequency responses of $\dot{\theta}/\delta$ and $\dot{\gamma}/\delta$ then were computed from these expressions for the mean flight condition of a Mach number of 2, an altitude of 30,000 feet, and a static margin of 25 percent of the mean aerodynamic chord of the isolated wing. The values of the derivatives for these computations and others given subsequently were obtained or extrapolated from available wind-tunnel data (references 1 and 2) or from theory (references 3, 4, and 5). These derivatives were assumed to be unaffected by frequency. The results of these computations are plotted in figure 5 in the usual form of curves of amplitude ratio and phase angle as functions of angular frequency. The solid curves are the results obtained by using all twenty derivatives. The dashed curves summarize the results of neglecting various derivatives or portions of a derivative until only the six indicated in the key remained. Since the difference between the two curves is almost indistinguishable, the six derivatives can be used to compute the frequency response adequately. A further reduction in their number cannot be made without a significant change in the dashed curves. The results for this example are also consistent with the simplification often employed in dynamic analyses in which only these same derivatives are used. A similar analysis was also made for a static margin of 5 percent with the same results.

Figure 6 is presented to illustrate the simplification in the frequency-response computations which results from a consideration of only the six important derivatives. In the figure are given the general differential equations relating the pitching velocity and the rate of change of the flight path to the control deflection when all the derivatives are considered and the complete expressions for the coefficients of the equations. The boxes enclose those terms composed of the six important derivatives, either singly or in combination, and the use of only those coefficients that are in boxes reduces the order of the polynomials in the general equations to a maximum of two. The resulting reduction in the complexity of the equations and the coefficients is shown in figure 7 in which the equations are written in a more fundamental form.

It must be realized that such simplifications must be applied with caution. Derivatives which may be unimportant under certain flight conditions may be significant for others. For example, one such case occurred in the present study where it was observed that the lift and pitching-moment coefficients per rate of change of control motion, that is, $C_{L_{\dot{\delta}}}$ and $C_{m_{\dot{\delta}}}$, may have some significance at a lower Mach number where the effect of the downwash lag on the tail is increased. Computations, therefore, were made at a Mach number of 1.2 at 30,000 feet and the results are shown in figure 8. The solid lines represent the results of using only the simplified equations with the six important derivatives, whereas the dashed lines include the effect of the derivatives $C_{L_{\dot{\delta}}}$ and $C_{m_{\dot{\delta}}}$. It appears that even at this Mach number the influence of $C_{L_{\dot{\delta}}}$ and $C_{m_{\dot{\delta}}}$ is small, except for the noticeable change in the phase angle of $\dot{\gamma}/\delta$ at the higher frequencies.

It is conceivable that marked changes in configuration would adjust the relative importance of the derivatives. However, an examination of the derivatives for a canard and a tail-control configuration indicated that the results are similar to those of the variable-incidence missile except that the lift derivative $C_{L_{\delta}}$ is of decreased importance.

The next question of interest concerns the effect on the frequency response of possible errors in evaluating the six significant derivatives. In studying these effects it was assumed that the lift derivatives could be evaluated to within 5 percent, the static-moment derivatives to 15 percent, and the rotary derivatives to within 20 percent. In figure 9 are shown the results of the computations of the frequency response. The solid line is identical to that given previously for a Mach number of 2 at 30,000 feet for a static margin of 25 percent and it was presumed that the derivatives were exact. The lowest broken line (lowest at the lower frequencies) labeled "accumulated error" was computed by using the possible errors in the derivatives in such a manner as to give the largest decrease in the amplitude ratio at zero frequency, henceforth referred to as the gearing, and the highest peak-amplitude ratio in relation to the steady-state value. The other broken curve is the result of using these errors conversely so that these two curves closely represent the limits in the variation in the frequency response due to the assumed possible inaccuracy of the derivatives. As can be seen the variation may be significant, amounting to, for example, a change in the gearing by as much as 28 percent. However, as is shown subsequently, other possible parameter variations have considerably greater effect.

Altitude and Mach Number

It is of interest to examine the changes in the frequency response which will be encountered due to variations in Mach number and altitude since a missile will normally be called upon to operate satisfactorily over relatively large ranges of these quantities. To determine their effect a Mach number range of 1.2 to 2.5 and a pressure-altitude range of 5,000 to 50,000 feet were selected as being representative limits of operation. The amplitude ratios and phase plots for these conditions are shown in figures 10 and 11. The effects are large, particularly that due to altitude. For example, for $\dot{\theta}/\delta$ the altitude change from 5,000 to 50,000 feet may change the gearing by a factor of 6-1/2, the frequency at the peak-amplitude ratio by almost 3, and the ratio of the peak amplitude to the steady-state value by as much as 5-1/2. In contrast, the Mach number effects are less than 2 for each of these comparisons. However, both of these effects are of a much greater magnitude than that due to inaccuracy in derivative evaluation.

Nonlinearities

Recent theoretical and experimental work has indicated that nonlinearities occur, particularly in the moment characteristics, due to the relative motion of the tail with respect to the vortices of the forward lifting surfaces. These nonlinearities are undesirable primarily because of the variations they introduce in the frequency response. Since their effect is greatest at a low Mach number and a high altitude where the largest angle-of-attack and control-deflection variation occurs, the assumed typical nonlinear pitching-moment characteristics at the lowest Mach number of 1.2 as shown in figure 12 were selected for examination. The trim points for zero lift and the assumed maximum allowable g at 50,000 feet are noted in the figure. If these are taken to be the limiting operating conditions of the missile, the maximum change in the frequency response due to the nonlinearity is closely given by using in the computations the slopes of the pitching-moment curves at these points. The effect on the frequency response of such variations in the static stability and control moment effectiveness is shown in figure 13. The change in the gearing at this flight condition is about 2-1/2 times the effect of Mach number for the linear case but only about half the variation introduced by altitude changes. The change in peak value relative to the steady-state amplitude ratio, however, is less than twice, and the frequency change is only slightly greater than that due to Mach number.

As mentioned previously, these results represent the most severe effect of nonlinearities. At other conditions of flight these effects are reduced because of the decreased angle-of-attack and control-deflection range.

Aeroelasticity

Since a premium is placed on weight in the design of a missile and the missile may be operated at extremely high dynamic pressure during a portion of its flight, the possibility of aeroelastic effects should be scrutinized. Theoretically, aeroelasticity introduces additional degrees of freedom to the equations of motion of the missile. However, if the natural frequency at zero airspeed of the missile components, such as the wing or tail, is many times that of the rigid-missile short-period natural frequency, the dynamics of the components will have little effect on the motions of the missile so that aeroelasticity effects may be treated as modifications to the usual significant static and rotary derivatives. To determine whether this may be the case for this missile, computations were made at the highest dynamic pressure considering the missile material to be duralumin. These computations indicated that the ratios of the natural frequencies of the components at zero airspeed to the short-period natural frequency of the rigid missile are approximately 25 each for the wing and body and 17 for the tail. This result may be indicative of the characteristics of most maneuvering-type missiles which normally employ low-aspect-ratio surfaces.

The effect of aeroelasticity, therefore, was computed for all four flight conditions neglecting the dynamics of the wing, tail, and body, and the results are presented in figures 14 and 15. It is seen that aeroelasticity tends to narrow the differences in the natural frequency. This is a favorable effect. However, the differences in the gearing are increased, which is unfavorable. This can be traced to the influence of the flexibility of the tail. For the condition of the highest dynamic pressure ($M = 2.5$ at an altitude of 5,000 ft) the changes to the frequency response due to aeroelasticity are somewhat less than those of the assumed nonlinearity at $M = 1.2$ and an altitude of 50,000 feet.

CONCLUSIONS

To conclude, the study of the effect of various factors on the frequency response of two important aerodynamic functions for the example variable-incidence supersonic missile may be summarized as follows:

1. The most significant aerodynamic parameters are the lift derivatives C_{L_α} and C_{L_δ}, the static-stability derivative C_{m_α}, the control-moment effectiveness derivative C_{m_δ}, and the damping derivatives $C_{m_{\dot\theta}}$ and $C_{m_{\dot\alpha}}$. The lift and moment derivatives due to the rate of change of control deflection $C_{L_{\dot\delta}}$ and $C_{m_{\dot\delta}}$ may have a significant effect on the phase of $\dot\gamma/\delta$ at low Mach numbers and high frequencies.

2. The effects of altitude on the frequency response are large, with Mach number changes resulting in a considerably smaller effect and both of these being much greater than that due to possible errors in the evaluation of the derivatives.

3. The greatest effect on the frequency response of the assumed nonlinearity in the pitching-moment coefficient with angle of attack and control deflection varied from being slightly greater to 2-1/2 times the effect of Mach number, depending on the frequency-response characteristic being compared.

4. The additional degrees of freedom due to aeroelasticity of components of the missile may normally be neglected and the aeroelastic effects taken into account by modifying the static and rotary derivatives. For the conditions considered, the greatest effect of aeroelasticity was somewhat less than that of the assumed nonlinearity. It also appears that aeroelasticity may be used to narrow variations of certain fundamental characteristics in the frequency response.

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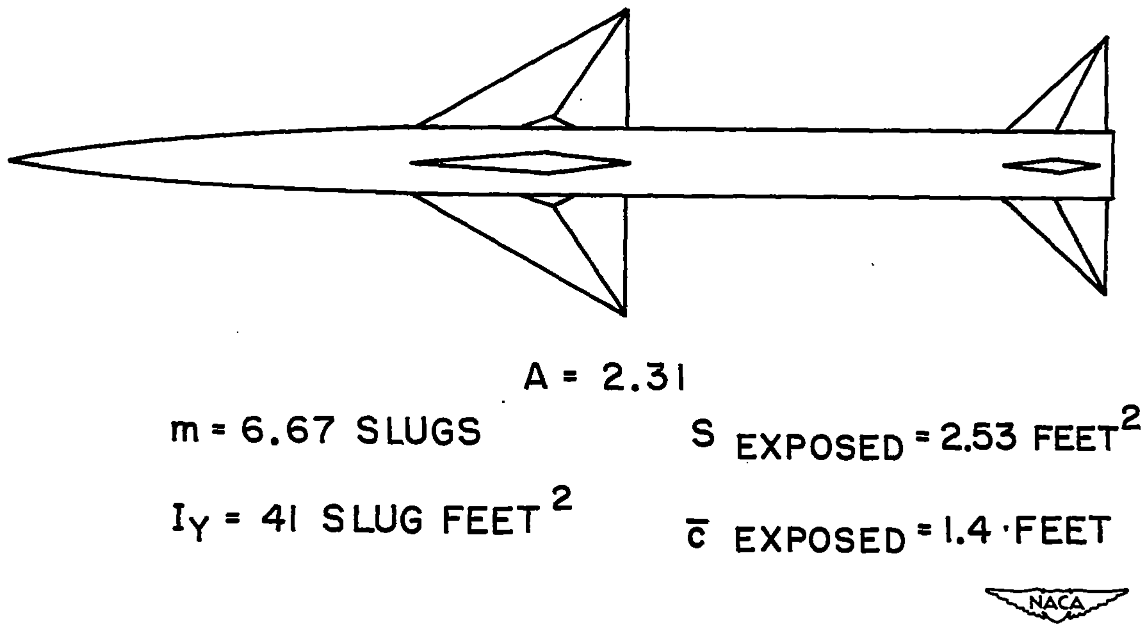


Figure 1.- Variable-incidence cruciform missile.

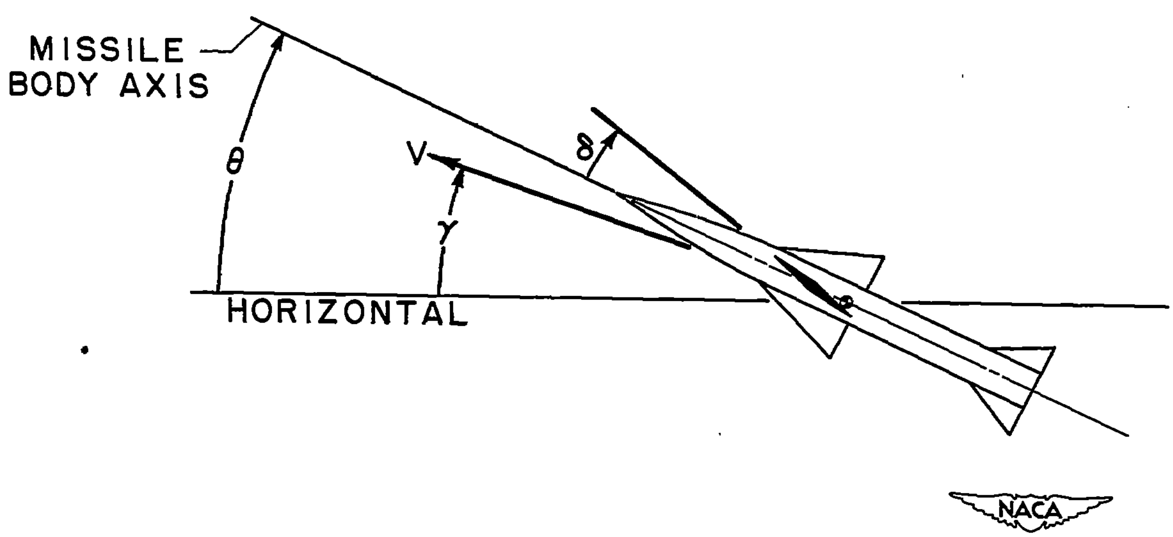


Figure 2.- Schematic diagram showing variables involved in missile motion.

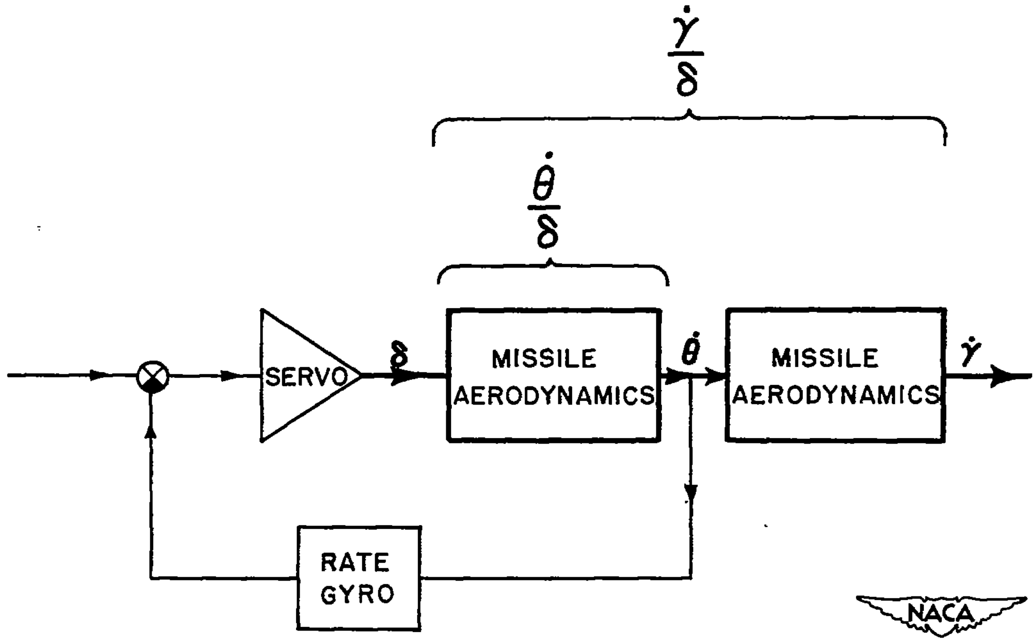
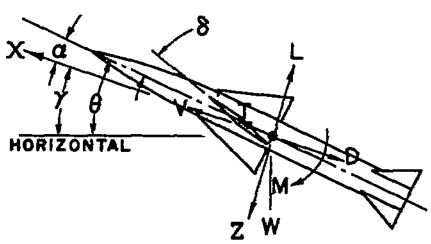


Figure 3.- Block diagram showing location of $\frac{\dot{\theta}}{\delta}$ and $\frac{\dot{\gamma}}{\delta}$.



$$(X_u+\tau D)u + (X_\alpha+DX_{\dot{\alpha}})\alpha + (X_\theta+DX_{\dot{\theta}})\theta = -(X_\delta+DX_{\dot{\delta}})\delta$$

$$Z_u u + [Z_\alpha+D(\tau+Z_{\dot{\alpha}})]\alpha + [Z_\theta-D(\tau-Z_{\dot{\theta}})]\theta = -(Z_\delta+DZ_{\dot{\delta}})\delta$$

$$C_{m_u}u + (C_{m_\alpha}+DC_{m_{\dot{\alpha}}})\alpha + (DC_{m_{\dot{\theta}}}-\sigma D^2)\theta = -(C_{m_\delta}+DC_{m_{\dot{\delta}}})\delta$$

– where –

$\tau = \frac{m}{\rho V s}$ $\sigma = \frac{I_y}{qS\bar{c}}$ $u = \frac{\Delta V}{V}$ $D = \frac{d}{dt}$

$X_u = C_D + \frac{V}{2}\frac{\partial C_D}{\partial V}$ $Z_u = C_L + \frac{V}{2}\frac{\partial C_L}{\partial V}$ $C_{m_u} = V\frac{\partial C_m}{\partial V}$

$X_\alpha = -\frac{1}{2}(C_L - C_{D_\alpha})$ $Z_\alpha = \frac{1}{2}(C_{L_\alpha} + C_D)$ $C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$

$X_\theta = \frac{1}{2}(C_L + C_T\alpha)$ $Z_\theta = -\frac{1}{2}(C_D - C_T)$ $C_{m_{\dot{\alpha}}} = \frac{\partial C_m}{\partial \dot{\alpha}}$

$X_{\dot{\alpha}} = \frac{C_{D_{\dot{\alpha}}}}{2}$ $Z_{\dot{\alpha}} = \frac{C_{L_{\dot{\alpha}}}}{2}$ $C_{m_{\dot{\theta}}} = \frac{\partial C_m}{\partial \dot{\theta}}$

$X_{\dot{\theta}} = \frac{C_{D_{\dot{\theta}}}}{2}$ $Z_{\dot{\theta}} = \frac{C_{L_{\dot{\theta}}}}{2}$ $C_{m_\delta} = \frac{\partial C_m}{\partial \delta}$

$X_\delta = \frac{C_{D_\delta}}{2}$ $Z_\delta = \frac{C_{L_\delta}}{2}$ $C_{m_{\dot{\delta}}} = \frac{\partial C_m}{\partial \dot{\delta}}$

$X_{\dot{\delta}} = \frac{C_{D_{\dot{\delta}}}}{2}$ $Z_{\dot{\delta}} = \frac{C_{L_{\dot{\delta}}}}{2}$

Figure 4.- Longitudinal equations of motion (constant thrust).

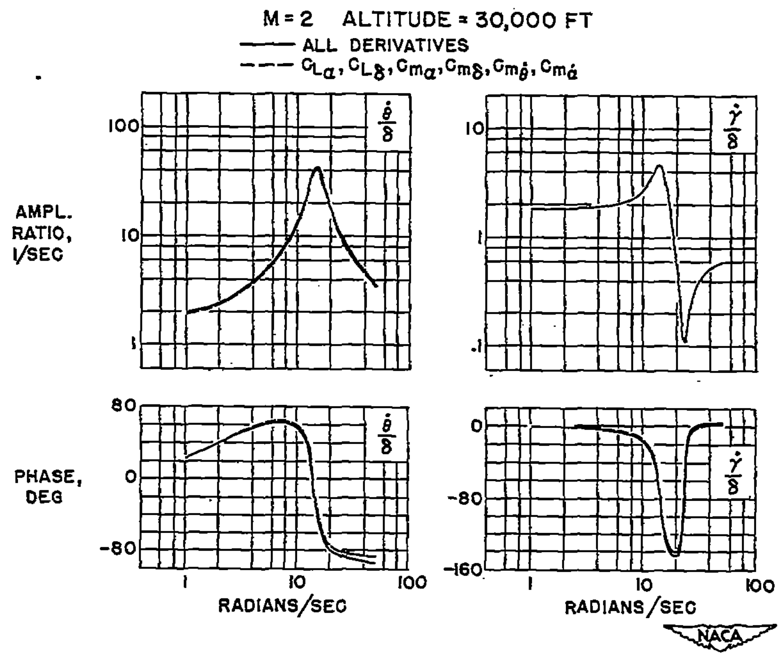


Figure 5.- Effect on frequency response of neglecting all but important derivatives.

$$\frac{\dot\theta}{\delta} = \frac{b_0 D + B_1 D^2 + B_2 D^3 + b_3 D^4}{a_0 + a_1 D + A_2 D^2 + A_3 D^3 + A_4 D^4}$$

$$\frac{\dot\gamma}{\delta} = \frac{c_0 D + C_1 D^2 + C_2 D^3 + C_3 D^4 + c_4 D^5}{a_0 + a_1 D + A_2 D^2 + A_3 D^3 + A_4 D^4}$$

- where -

$$\begin{aligned}
a_0 &= C_{m_\alpha}(X_\theta Z_u - X_u Z_\theta) + C_{m_u}(X_\alpha Z_\theta - X_\theta Z_\alpha) \\
a_1 &= C_{m_{\dot\alpha}}(X_\theta Z_u - X_u Z_\theta) + C_{m_{\dot\theta}}(X_u Z_\alpha - X_\alpha Z_u) + \\
&\quad C_{m_\alpha}[(X_{\dot\theta} Z_u - X_u Z_{\dot\theta}) + \tau(X_u - Z_\theta)] + \\
&\quad C_{m_u}[(X_\alpha Z_{\dot\theta} - X_{\dot\theta} Z_\alpha) + (X_{\dot\alpha} Z_\theta - X_\theta Z_{\dot\alpha}) - \tau(X_\alpha + X_\theta)] \\
A_2 &= \boxed{\tau(\tau C_{m_\alpha} + Z_\alpha C_{m_{\dot\theta}})} - C_{m_{\dot\alpha}}[(X_u Z_{\dot\theta} - X_{\dot\theta} Z_u) + \tau(Z_\theta - X_u)] + \\
&\quad C_{m_{\dot\theta}}[(X_u Z_{\dot\alpha} - X_{\dot\alpha} Z_u) + \tau X_u] + \\
&\quad C_{m_u}[(X_{\dot\alpha} Z_{\dot\theta} - X_{\dot\theta} Z_{\dot\alpha}) - \tau(X_{\dot\alpha} + X_{\dot\theta})] - \\
&\quad \tau C_{m_\alpha} Z_{\dot\theta} + \sigma(X_\alpha Z_u - X_u Z_\alpha) \\
A_3 &= \boxed{\tau[\tau(C_{m_{\dot\alpha}} + C_{m_{\dot\theta}}) - \sigma Z_\alpha]} + \tau(C_{m_{\dot\theta}} Z_{\dot\alpha} - C_{m_{\dot\alpha}} Z_{\dot\theta}) - \\
&\quad \sigma[(X_u Z_{\dot\alpha} - X_{\dot\alpha} Z_u) + \tau X_u] \\
A_4 &= \boxed{-\sigma\tau^2} - \sigma\tau Z_{\dot\alpha} \\
b_0 &= C_{m_\delta}(X_\alpha Z_u - X_u Z_\alpha) + C_{m_\alpha}(X_u Z_\delta - X_\delta Z_u) + \\
&\quad C_{m_u}(X_\delta Z_\alpha - X_\alpha Z_\delta)
\end{aligned}$$

$$\begin{aligned}
B_1 &= \boxed{\tau(C_{m_\alpha} Z_\delta - C_{m_\delta} Z_\alpha)} + C_{m_{\dot\delta}}(X_\alpha Z_u - X_u Z_\alpha) + \\
&\quad C_{m_{\dot\alpha}}(X_u Z_\delta - X_\delta Z_u) + C_{m_\alpha}(X_u Z_{\dot\delta} - X_{\dot\delta} Z_u) - C_{m_\delta}[(X_u Z_{\dot\alpha} - X_{\dot\alpha} Z_u) + \tau X_u] + \\
&\quad C_{m_u}[(X_\delta Z_{\dot\alpha} - X_{\dot\alpha} Z_\delta) + (X_{\dot\delta} Z_\alpha - X_\alpha Z_{\dot\delta}) + \tau X_\delta] \\
B_2 &= \boxed{-\tau(\tau C_{m_\delta} - C_{m_{\dot\alpha}} Z_\delta)} - C_{m_{\dot\delta}}[(X_u Z_{\dot\alpha} - X_{\dot\alpha} Z_u) + \tau(X_u + Z_\alpha)] + \\
&\quad C_{m_{\dot\alpha}}(X_u Z_{\dot\delta} - X_{\dot\delta} Z_u) + C_{m_u}[(X_{\dot\delta} Z_{\dot\alpha} - X_{\dot\alpha} Z_{\dot\delta}) + \tau X_{\dot\delta}] + \\
&\quad \tau(C_{m_\alpha} Z_{\dot\delta} - C_{m_\delta} Z_{\dot\alpha}) \\
b_3 &= \tau(C_{m_{\dot\alpha}} Z_{\dot\delta} - C_{m_{\dot\delta}} Z_{\dot\alpha}) - \tau^2 C_{m_{\dot\delta}} \\
c_0 &= C_{m_\delta}[(X_\alpha Z_u - X_u Z_\alpha) + (X_\theta Z_u - X_u Z_\theta)] + C_{m_\alpha}(X_u Z_\delta - X_\delta Z_u) + \\
&\quad C_{m_u}[(X_\delta Z_\alpha - X_\alpha Z_\delta) + (X_\delta Z_\theta - X_\theta Z_\delta)] \\
C_1 &= \boxed{-\tau(C_{m_\delta} Z_\alpha - C_{m_\alpha} Z_\delta)} + C_{m_{\dot\delta}}[(X_\alpha Z_u - X_u Z_\alpha) + (X_\theta Z_u - X_u Z_\theta)] + \\
&\quad (C_{m_{\dot\alpha}} + C_{m_{\dot\theta}})(X_u Z_\delta - X_\delta Z_u) + C_{m_\alpha}(X_u Z_{\dot\delta} - X_{\dot\delta} Z_u) - \\
&\quad C_{m_\delta}[(X_u Z_{\dot\alpha} - X_{\dot\alpha} Z_u) + (X_u Z_{\dot\theta} - X_{\dot\theta} Z_u) + \tau Z_\theta] + \\
&\quad C_{m_u}[(X_{\dot\delta} Z_\alpha - X_\alpha Z_{\dot\delta}) + (X_\delta Z_{\dot\alpha} - X_{\dot\alpha} Z_\delta) + (X_{\dot\delta} Z_\theta - X_\theta Z_{\dot\delta}) + (X_\delta Z_{\dot\theta} - X_{\dot\theta} Z_\delta)] \\
C_2 &= \boxed{\tau Z_\delta(C_{m_{\dot\alpha}} + C_{m_{\dot\theta}})} - C_{m_{\dot\delta}}[(X_u Z_{\dot\alpha} - X_{\dot\alpha} Z_u) + (X_u Z_{\dot\theta} - X_{\dot\theta} Z_u) + \tau(Z_\alpha + Z_\theta)] + \\
&\quad (C_{m_{\dot\alpha}} + C_{m_{\dot\theta}})(X_u Z_{\dot\delta} - X_{\dot\delta} Z_u) + C_{m_u}[(X_{\dot\delta} Z_{\dot\alpha} - X_{\dot\alpha} Z_{\dot\delta}) + (X_{\dot\delta} Z_{\dot\theta} - X_{\dot\theta} Z_{\dot\delta})] + \\
&\quad \tau[(C_{m_\alpha} Z_{\dot\delta} - C_{m_\delta} Z_{\dot\alpha}) - C_{m_\delta} Z_{\dot\theta}] - \sigma(X_u Z_\delta - X_\delta Z_u) \\
C_3 &= \boxed{-\sigma\tau Z_\delta} + \tau Z_{\dot\delta}(C_{m_{\dot\alpha}} + C_{m_{\dot\theta}}) - \tau C_{m_{\dot\delta}}(Z_{\dot\alpha} + Z_{\dot\theta}) - \sigma(X_u Z_{\dot\delta} - X_{\dot\delta} Z_u) \\
c_4 &= -\sigma\tau Z_{\dot\delta}
\end{aligned}$$

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Figure 6.- Complete expressions for $\frac{\dot\theta}{\delta}$ and $\frac{\dot\gamma}{\delta}$.

$$\frac{\dot{\theta}}{\delta} = \frac{K_{\dot{\theta}}(1+T_{\dot{\theta}}D)}{\left(1 + \frac{2\zeta}{\omega_n}D + \frac{D^2}{\omega_n^2}\right)} \qquad \frac{\dot{\gamma}}{\delta} = \frac{K_{\dot{\gamma}}\left(1 + \frac{2\zeta_{\dot{\gamma}}}{\omega_{\dot{\gamma}}}D + \frac{D^2}{\omega_{\dot{\gamma}}^2}\right)}{\left(1 + \frac{2\zeta}{\omega_n}D + \frac{D^2}{\omega_n^2}\right)}$$

— where —

$$K_{\dot{\theta}} = K_{\dot{\gamma}} = \frac{C_{L_\alpha}C_{m_\delta} - C_{L_\delta}C_{m_\alpha}}{-2\tau C_{m_\alpha} - C_{m_{\dot{\theta}}}C_{L_\alpha}} \qquad T_{\dot{\theta}} = \frac{2\tau C_{m_\delta} - C_{L_\delta}C_{m_{\dot{\alpha}}}}{C_{L_\alpha}C_{m_\delta} - C_{L_\delta}C_{m_\alpha}}$$

$$\zeta = \frac{1}{2\omega_n}\left(\frac{C_{L_\alpha}}{2\tau} - \frac{C_{m_{\dot{\theta}}} + C_{m_{\dot{\alpha}}}}{\sigma}\right) \qquad \zeta_{\dot{\gamma}} = \frac{-(C_{m_{\dot{\theta}}} + C_{m_{\dot{\alpha}}})}{2\sigma\omega_{\dot{\gamma}}}$$

$$\omega_n = \left(\frac{-2\tau C_{m_\alpha} - C_{m_{\dot{\theta}}}C_{L_\alpha}}{2\tau\sigma}\right)^{\frac{1}{2}} \approx \left(-\frac{C_{m_\alpha}}{\sigma}\right)^{\frac{1}{2}} \qquad \omega_{\dot{\gamma}} = \left(\frac{C_{L_\alpha}C_{m_\delta} - C_{L_\delta}C_{m_\alpha}}{\sigma C_{L_\delta}}\right)^{\frac{1}{2}}$$

Figure 7.- Simplified form of $\frac{\dot{\theta}}{\delta}$ and $\frac{\dot{\gamma}}{\delta}$.

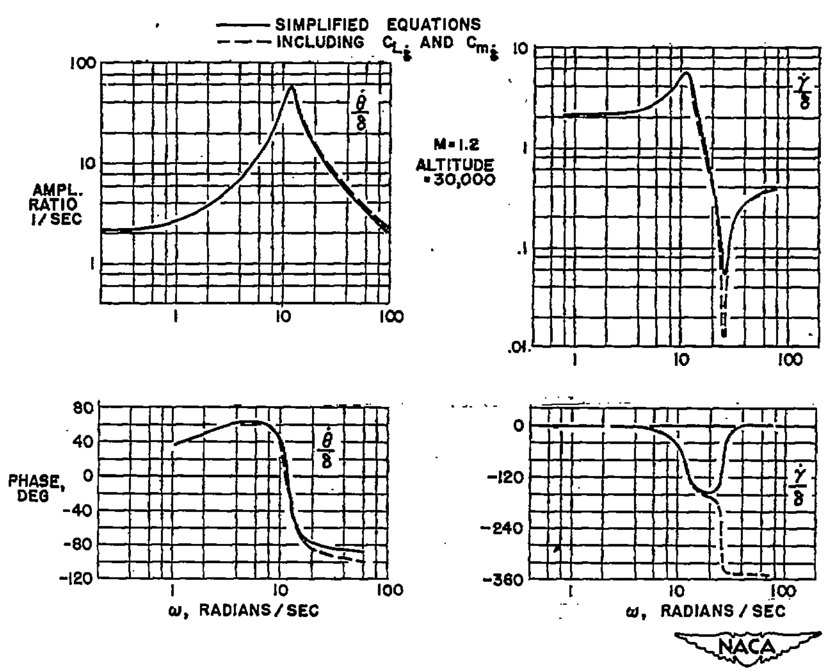

Figure 8.- Effect of $C_{L_{\dot{\delta}}}$ and $C_{m_{\dot{\delta}}}$.

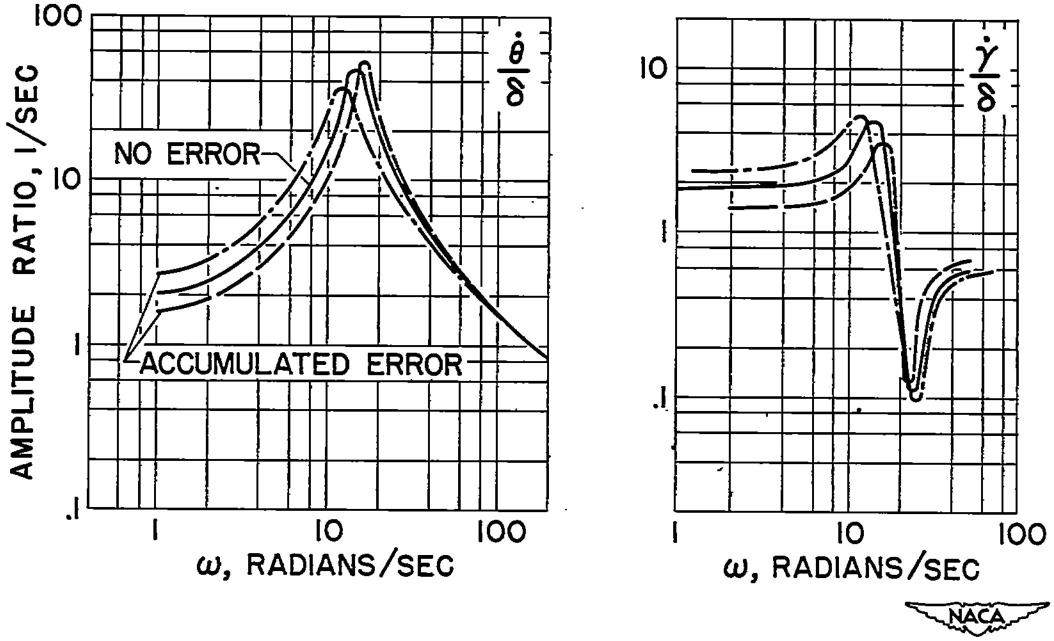

Figure 9.- Effect of derivative inaccuracy; M, 2.0; altitude, 30,000 feet.

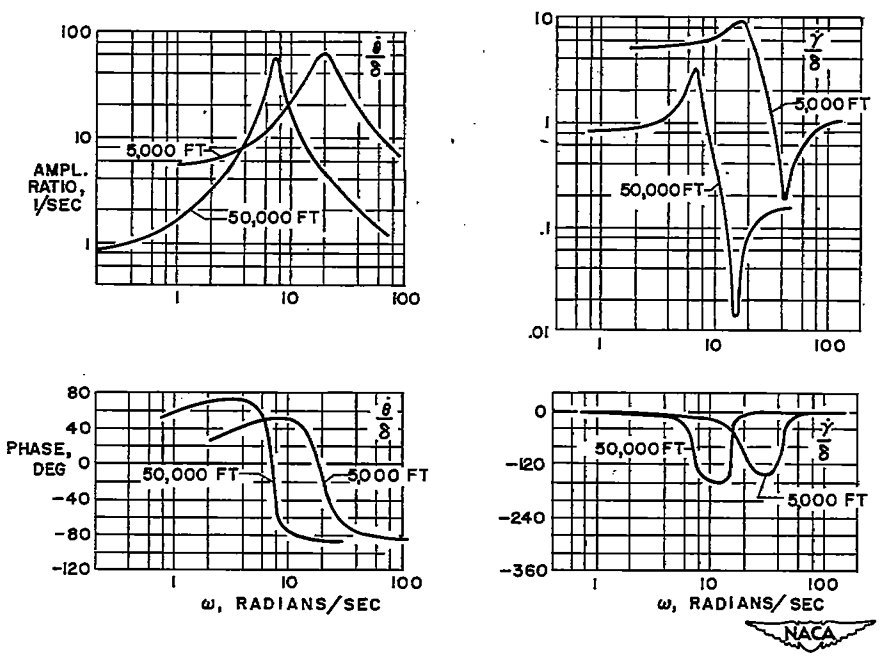

Figure 10.- Effect of altitude, M = 1.2.

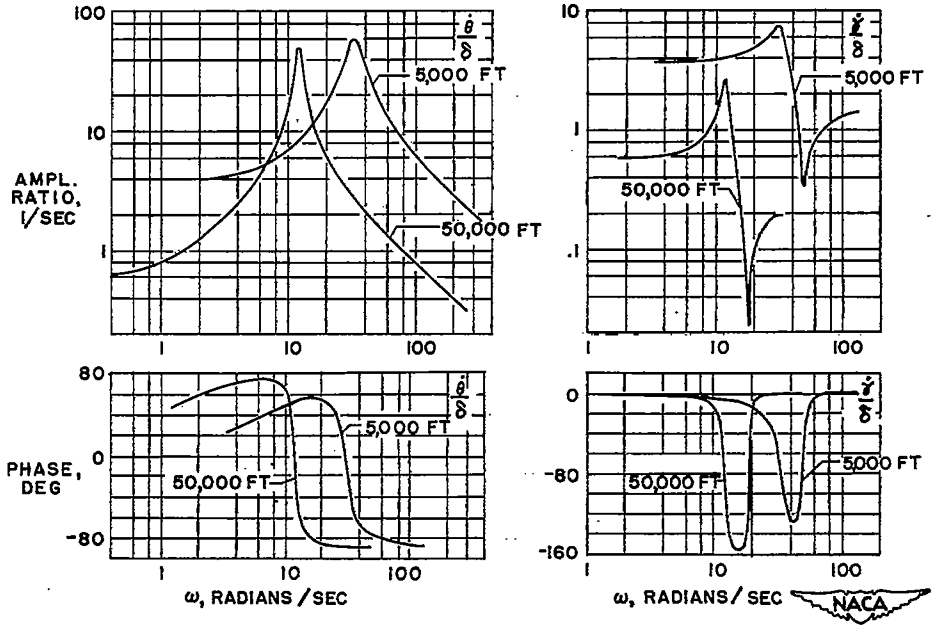

Figure 11.- Effect of altitude, M = 2.5.

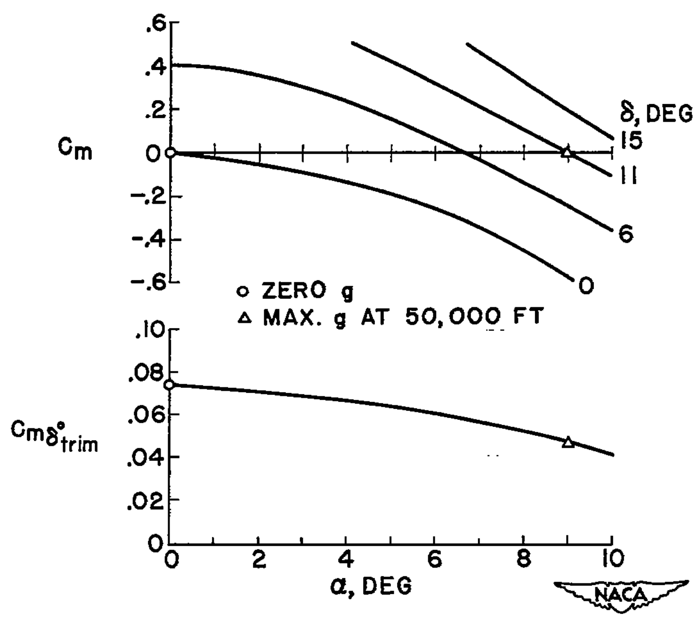

Figure 12.- Nonlinear variation of missile pitching moment with angle of attack, M = 1.2.

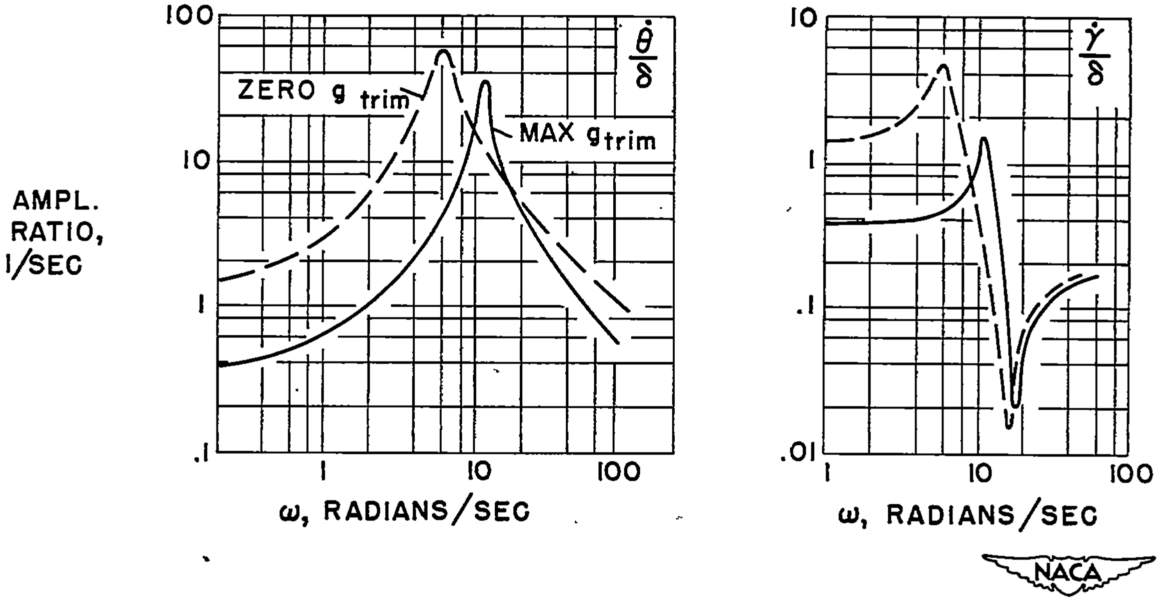

Figure 13.- Effect of nonlinear variation of pitching moment with angle of attack; M, 1.2; altitude, 50,000 feet.

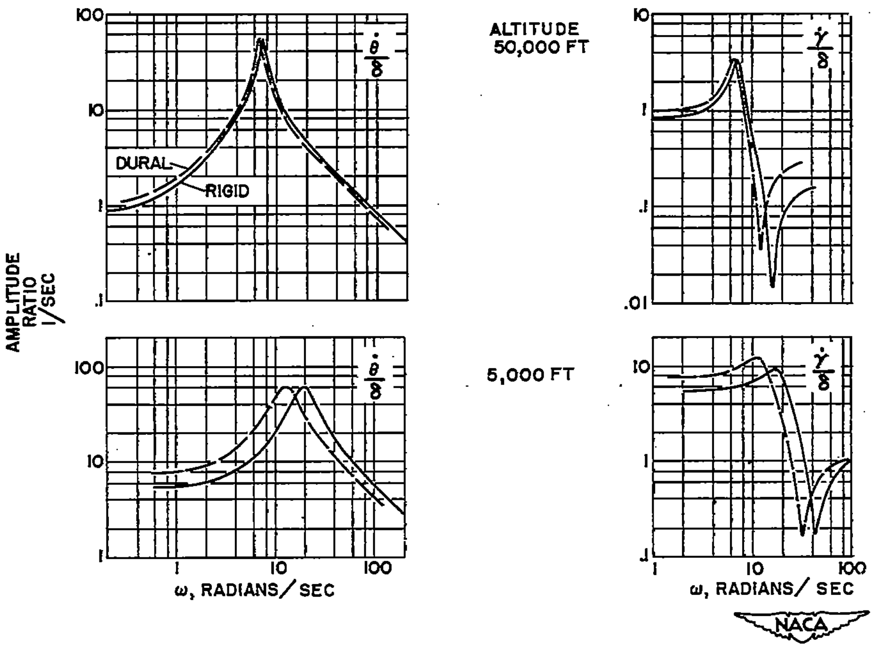

Figure 14.- Effect of aeroelasticity, M = 1.2.

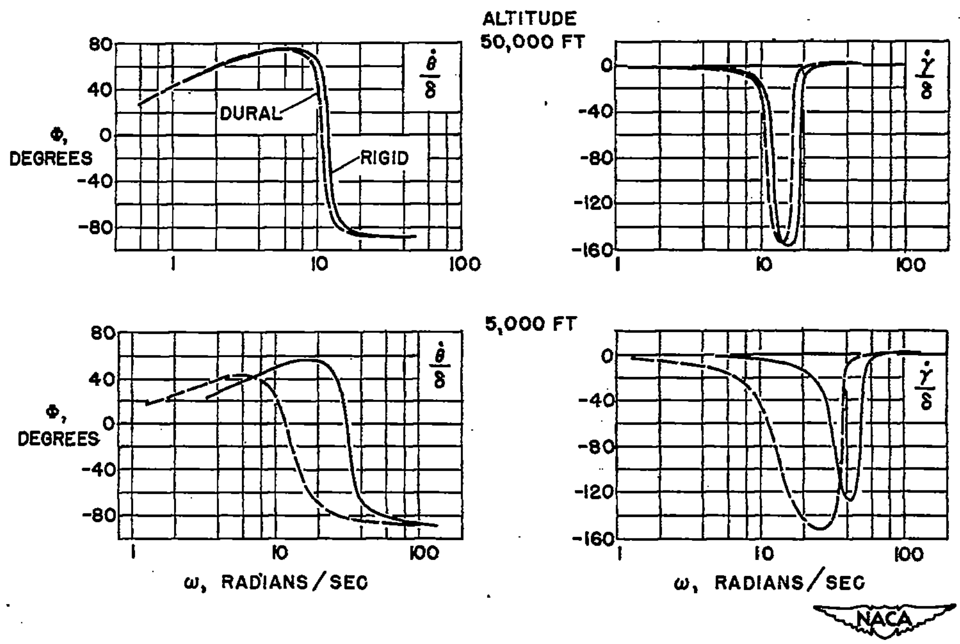

Figure 15.- Effect of aeroelasticity, M = 2.5.

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